

3<sup>rd</sup> Cuneo City ImmunoTherapy Conference (CCITC)

# Immunotherapy in Hematological Malignancies **2023**

CUNEO  
May 18-20, 2023

Spazio incontri Fondazione CRC

Organized by Prof. Massimo Massaia, SC Ematologia AO S.Croce e Carle, Cuneo, Italy  
and Centro Interdipartimentale di Ricerca in Biologia Molecolare (CIRBM), Torino, Italy

## Immunotherapy in Hematological Malignancies 2023

### DICHIARAZIONE

Relatore: TOMMASO LORENZI

Come da nuova regolamentazione della Commissione Nazionale per la Formazione Continua del Ministero della Salute, è richiesta la trasparenza delle fonti di finanziamento e dei rapporti con soggetti portatori di interessi commerciali in campo sanitario.

- Posizione di dipendente in aziende con interessi commerciali in campo sanitario **(NIENTE DA DICHIARARE)**
- Consulenza ad aziende con interessi commerciali in campo sanitario **(NIENTE DA DICHIARARE)**
- Fondi per la ricerca da aziende con interessi commerciali in campo sanitario **(NIENTE DA DICHIARARE)**
- Partecipazione ad Advisory Board **(NIENTE DA DICHIARARE)**
- Titolarità di brevetti in compartecipazione ad aziende con interessi commerciali in campo sanitario **(NIENTE DA DICHIARARE)**
- Partecipazioni azionarie in aziende con interessi commerciali in campo sanitario **(NIENTE DA DICHIARARE)**

# Dissecting Cancer Development and Response to Therapy Through Mathematics

A (very) short introduction through a few case studies

**Tommaso Lorenzi**

Department of Mathematical Sciences  
Politecnico di Torino



# Immunotherapy in Hematological Malignancies 2023

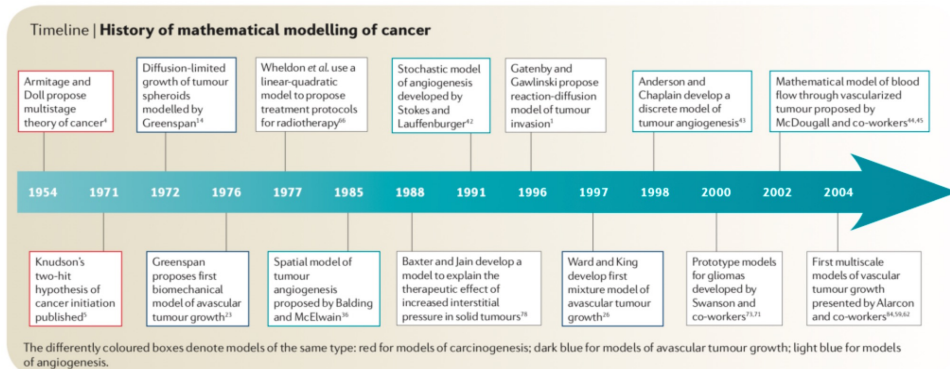
## Premises

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# Immunotherapy in Hematological Malignancies 2023

## Mathematical modelling of cancer

- The **mathematical modelling** of the processes that underpin **cancer dynamics** has been a **very active research area** in the last 70 years.



Adapted from Byrne, *Nat. Rev. Cancer.* 10:221–230, 2010.

- Cancer modelling has been a fruitful **source of knowledge transfer** and **mutual enrichment** **between mathematics and cancer research.**

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## Immunotherapy in Hematological Malignancies 2023

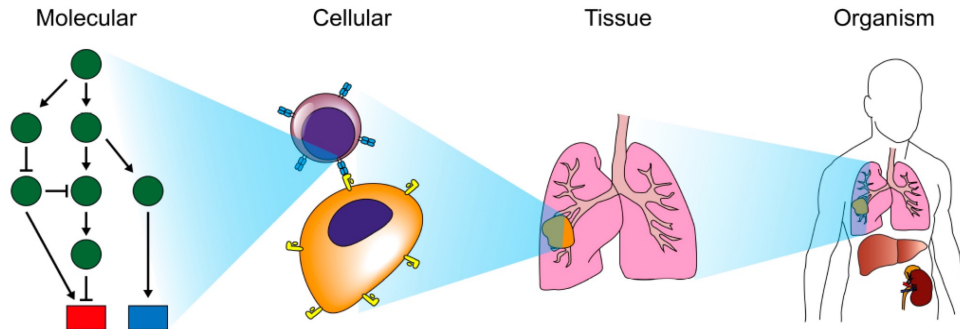
### Knowledge transfer between mathematics and cancer research

- Mathematics gives theoretical support to cancer research by:
  - ▶ offering a formal framework in which to consolidate the profusion of experimental and clinical data being generated;
  - ▶ providing models to virtually dissect cancer dynamics and carry out *in silico* investigations to identify targets of therapeutic intervention.
- Cancer research acts as a catalyst for mathematical progress by:
  - ▶ providing a fertile ground for mathematical challenges, which entail developing new mathematical methods and techniques;
  - ▶ stimulating interdisciplinary research that has promoted cross-fertilisation of multiple branches of the mathematical sciences.

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## Representation scales in cancer modelling

- A wide range of mathematical models for the mechanisms that underlie cancer dynamics at different biological scales has been developed.



Adapted from Reticker-Flynn and Engleman, eLife 2020;9:e53839, 2020.

- The focus of this talk is on mathematical models for the dynamics of phenomena that are observed at the cellular and tissue scales.

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### Types of mathematical models for complex phenomena

- When studying **complex phenomena**, such as those implicated in cancer, two main **types of mathematical models** can be employed:
  1. **more comprehensive models** that capture as many aspects and facts as possible  $\Rightarrow$  **more detailed description** but more parameters to estimate and **little mathematical tractability**  $\Rightarrow$  **less robust conclusions**;
  2. **less comprehensive models** that capture only salient aspects and facts  $\Rightarrow$  **less detailed description** but fewer parameters to estimate and **greater mathematical tractability**  $\Rightarrow$  **more robust conclusions**.
- **In this talk**, mathematical **models of the second type** that are formulated as **partial differential equations (PDEs)** will be considered.

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## PDE models in physics and biology

- Such PDE models have been used to describe **physical phenomena**.

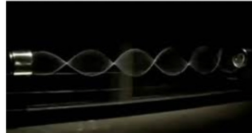
heat diffusion



$$\partial_t u - \Delta u = 0$$

(Fourier)

wave propagation



$$\partial_{tt}^2 u - \Delta u = 0$$

(D'Alembert)

gas dynamics

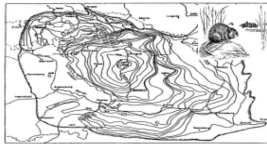


$$\partial_t f + \xi \cdot \nabla f = Q(f, f)$$

(Boltzmann)

- More recently, they have been used to describe **biological phenomena**.

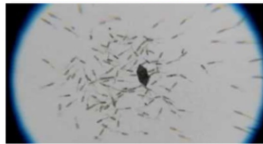
species invasion



$$\partial_t n - \Delta n = f(n)$$

(Fisher-Kolmogorov-  
Petrovskii-Piskunov)

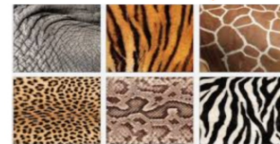
bacterial chemotaxis



$$\begin{cases} \partial_t n - \nabla \cdot (\nabla n - n \chi \nabla c) = 0 \\ \partial_t c - \Delta c = n - c \end{cases}$$

(Keller-Segel)

formation of animal patterns



$$\begin{cases} \partial_t u - D_u \Delta u = f(u, v) \\ \partial_t v - D_v \Delta v = g(u, v) \end{cases}$$

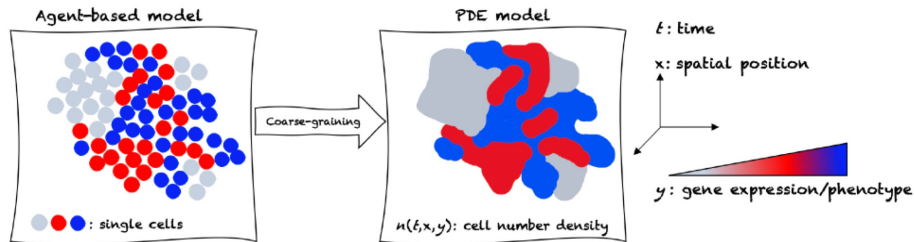
(Turing)

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### PDE models in cancer research

- My research interests center around **PDE models** for cellular processes that are implicated in the emergence and evolution of:
  - ▶ clonal heterogeneity in acute leukemias and intra-tumour heterogeneity;
  - ▶ resistance to anti-cancer drugs and adaptive immune resistance;
  - ▶ cancer invasion and metastasis.
- These models describe the **evolutionary & spatial dynamics of cell populations** and can be derived through the **coarse-graining of agent-based models** (i.e. models that track dynamics of single cells).



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## Case Study I

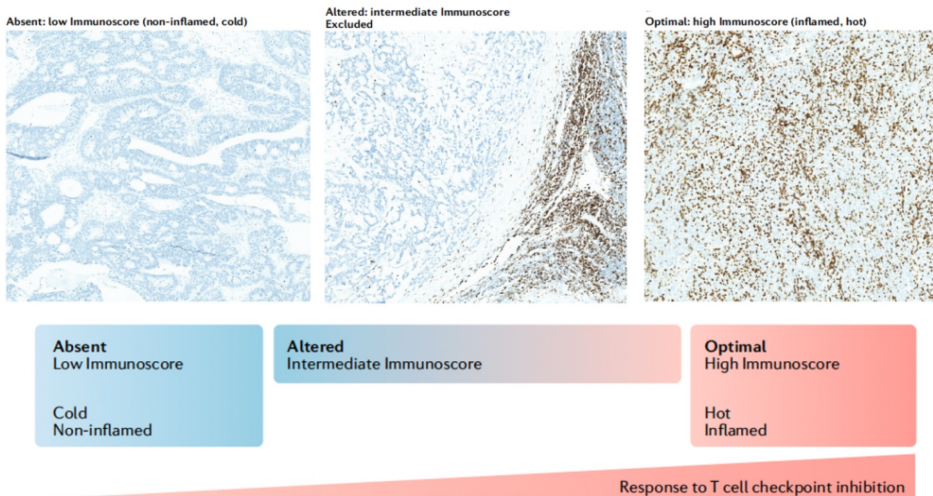
Mathematical models for studying  
immune infiltration into solid tumours and the response to immunotherapy

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# Immunotherapy in Hematological Malignancies 2023

## Background

### Immune cell infiltration into solid tumours

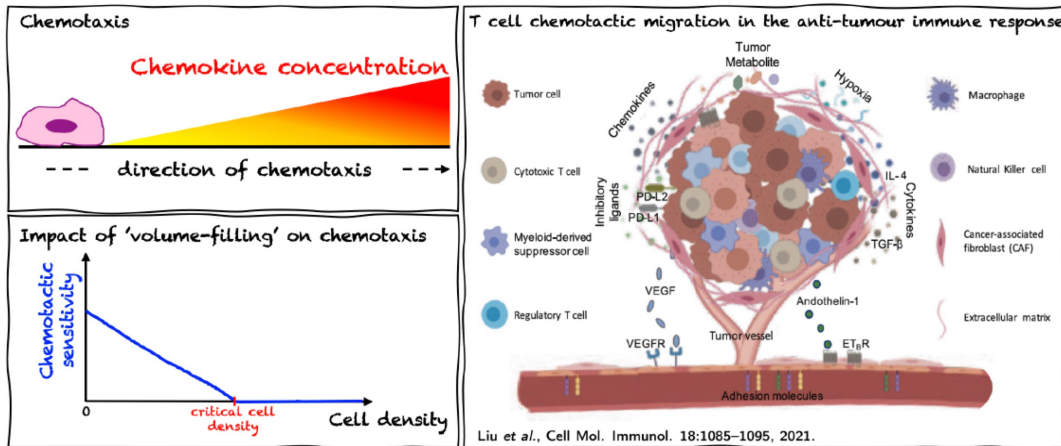


Adapted from Galon and Bruni, Nat. Rev. Drug Discov. 18:197–218, 2019.

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## Background

### T cell chemotactic migration in the anti-tumour immune response



T. Lorenzi (Politecnico di Torino)

## Research question

How do the governing parameters of T cell chemotactic migration impact on immune infiltration and on the response to anti-cancer immunotherapy?

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## Mathematical model

$n(t, x)$  : tumour cell density     $c(t, x)$  : density of activated CD8<sup>+</sup> T cells

$\phi(t, x)$  : concentration of chemokines

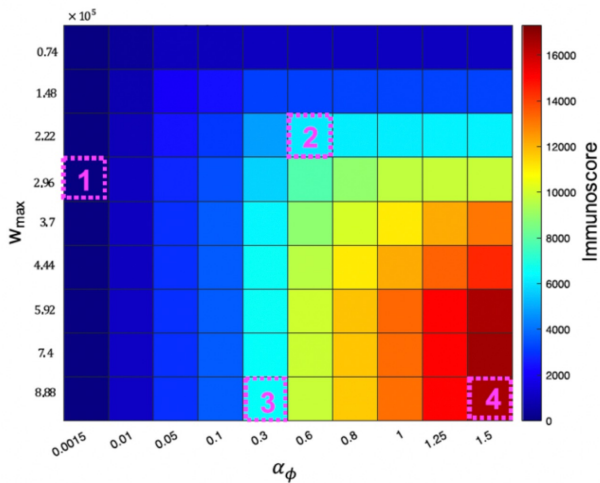
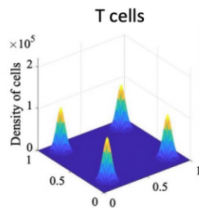
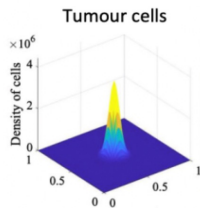
$$\left\{ \begin{array}{l} \partial_t n = \underbrace{(\alpha_n - \mu_n N) n}_{\text{proliferation and death}} - \underbrace{\zeta_n K(c) n}_{\text{death caused by T cells}} \\ \partial_t c - \underbrace{\nabla \cdot (\beta_c \psi(\rho) \nabla c - c \gamma_c \psi(\rho) \nabla \rho - c \beta_c \psi'(\rho) \nabla \rho)}_{\text{migration}} = \underbrace{\Sigma(t, x) - M(C) c}_{\text{immune regulation}} \\ \partial_t \phi - \underbrace{\beta_\phi \Delta \phi}_{\text{diffusion}} = \underbrace{\alpha_\phi n - \kappa_\phi \phi}_{\text{secretion and decay}} \end{array} \right. \quad x \in \Omega$$

$$\rho(t, x) := n(t, x) + c(t, x), \quad N(t) := \int_{\Omega} n(t, x) dx, \quad C(t) := \int_{\Omega} c(t, x) dx$$

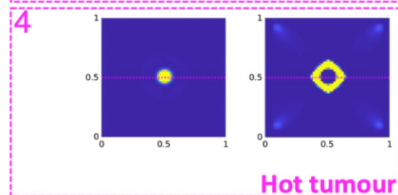
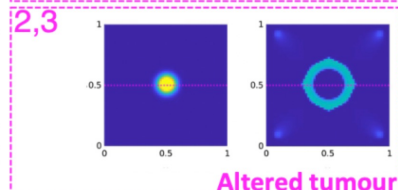
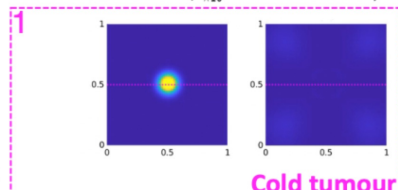
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# Immunotherapy in Hematological Malignancies 2023

## Main results



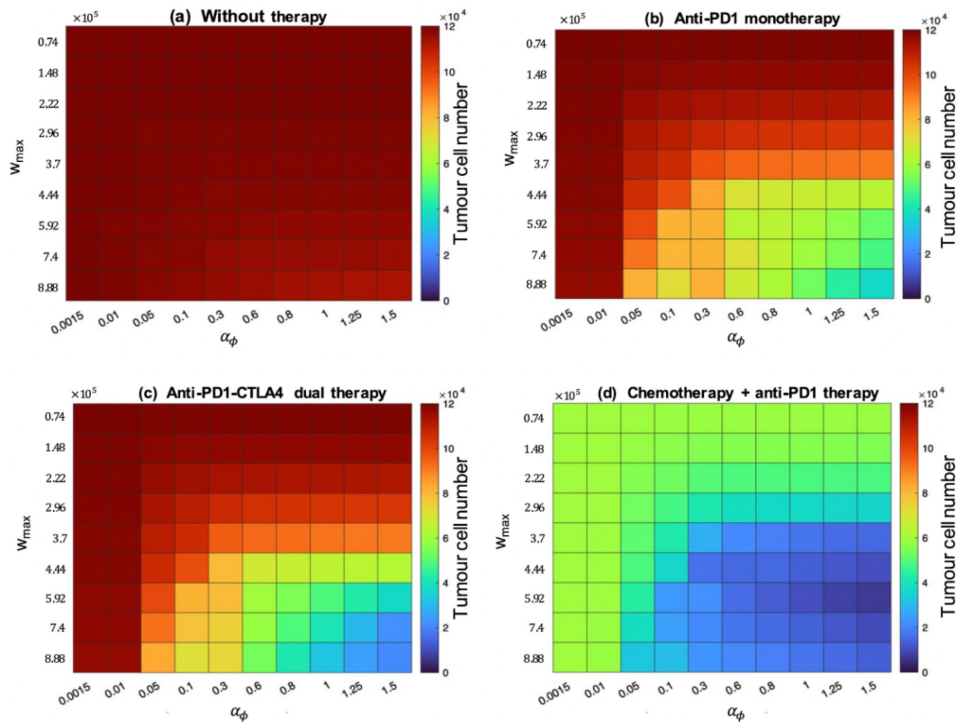
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# Immunotherapy in Hematological Malignancies 2023

## Main results



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## Case Study II

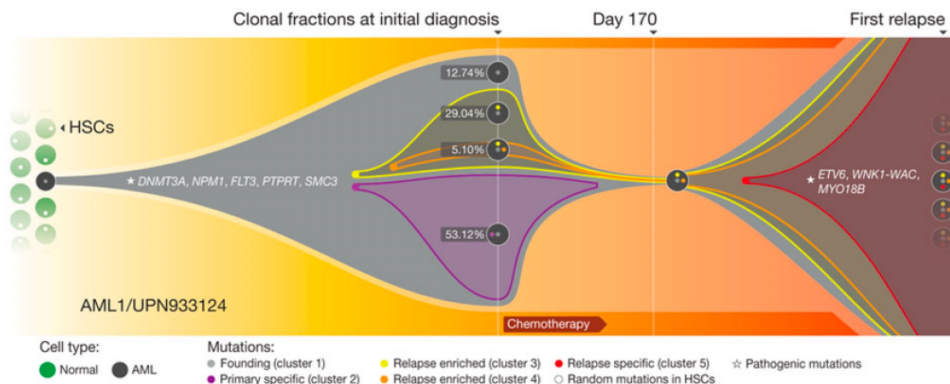
Mathematical models for studying  
clonal heterogeneity in acute leukemias

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# Immunotherapy in Hematological Malignancies 2023

## Background

### Clonal heterogeneity in acute leukemias

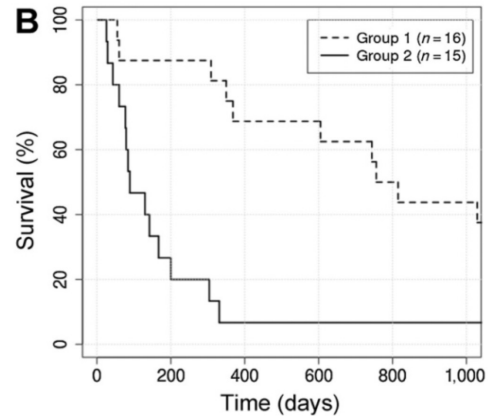
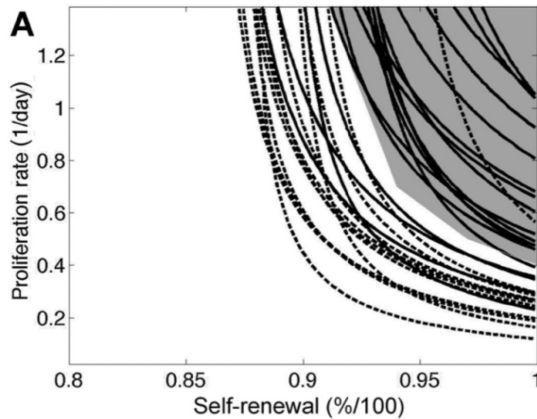


Ding *et al.*, Nature 481:506–510, 2012.

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## Background

Proliferation rate and self-renewal fraction of leukemic cells affect course of disease



Stiehl *et al.*, Cancer Res. 75:940–949, 2015.

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## Research question

How do the proliferation rate and the self-renewal fraction of leukemic cells shape the development of clonal heterogeneity in acute leukemias?

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## Mathematical model

$n_i(t, y)$ : number density of leukemic cells of clone  $y \in (0, 1]$  at maturation stage  $i$

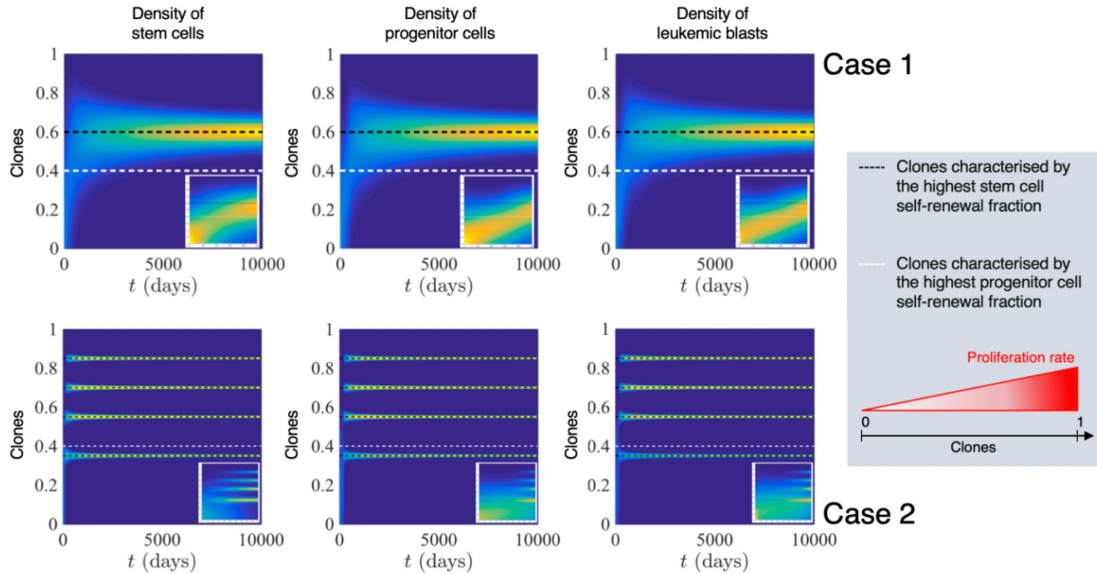
$i = 1$ : stem cells       $i = 2, \dots, M-1$ : progenitor cells       $i = M$ : leukemic blasts

$$\left\{ \begin{array}{l} \partial_t n_1 = \underbrace{\left( \frac{2a_1(y)}{1+K\rho_M} - 1 \right)}_{\text{self-renewal}} p_1(y) n_1 \\ \partial_t n_i = 2 \underbrace{\left( 1 - \frac{a_{i-1}(x)}{1+K\rho_M(t)} \right)}_{\substack{\text{differentiation of cells} \\ \text{at maturation stage } i-1}} p_{i-1}(y) n_{i-1} \\ \quad + \underbrace{\left( \frac{2a_i(y)}{1+K\rho_M} - 1 \right)}_{\text{self-renewal}} p_i(x) n_i, \quad i = 2, \dots, M-1 \quad y \in (0, 1] \\ \partial_t n_M = 2 \underbrace{\left( 1 - \frac{a_{M-1}(y)}{1+K\rho_M} \right)}_{\substack{\text{differentiation of cells} \\ \text{at maturation stage } M-1}} p_{M-1}(y) n_{M-1} - \underbrace{d n_M}_{\text{death}} \\ \rho_M(t) := \int_0^1 n_M(t, y) dy \end{array} \right.$$

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# Immunotherapy in Hematological Malignancies 2023

## Main results



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## Case Study III

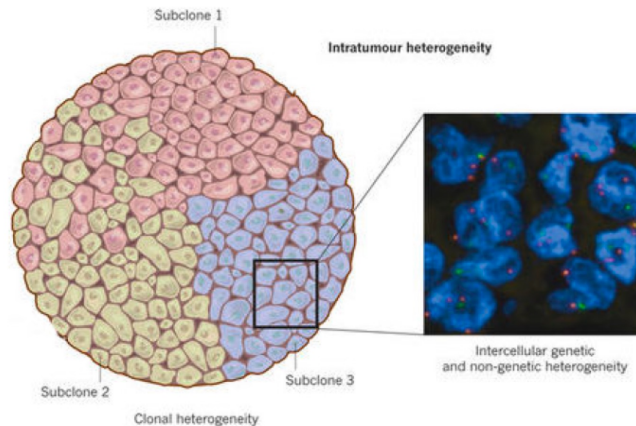
Mathematical models for studying  
intra-tumour heterogeneity in vascularised tumours

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## Background

### Intra-tumour heterogeneity

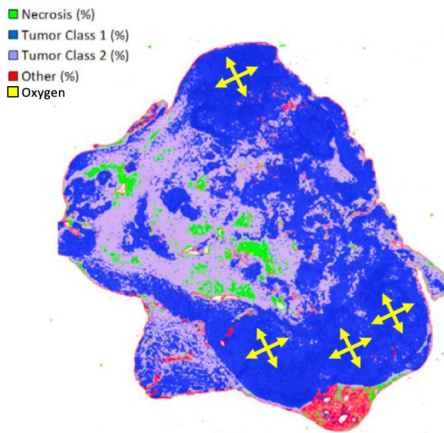


Adapted from Burrell *et al.*, Nature 501:338–345, 2013.

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## Background

Links between cell characteristics and oxygen distribution in vascularised tumours



Adapted from Gallaher *et al.*, *Sci. Rep.* 9:2425, 2019.

- oxygenated regions → oxidative phenotype
- hypoxic regions → hypoxic phenotype

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## Research question

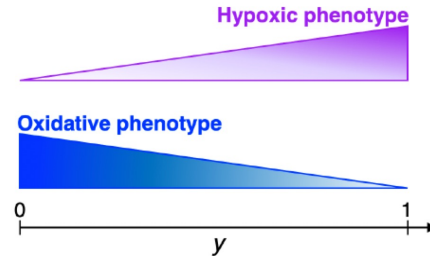
Can we explain such experimental observations  
as the outcome of adaptation of tumour cells  
to the spatial distribution of oxygen?

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## Mathematical model

$n(t,x,y)$  : density of tumour cells  
in metabolic state  $y$



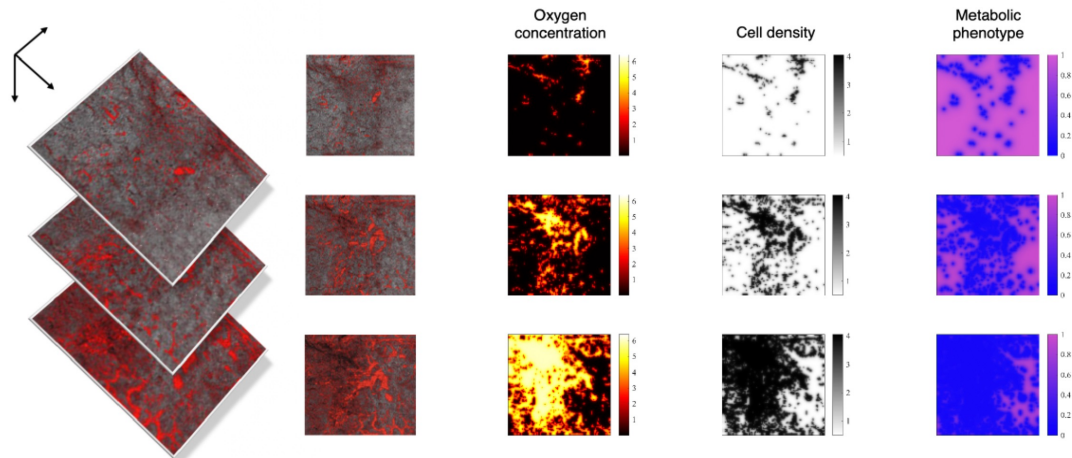
$s(t,x)$  : oxygen concentration

$$\left\{ \begin{array}{l} \partial_t n - \underbrace{\alpha \Delta_x n}_{\text{movement}} = \underbrace{(r(s,y) - \zeta \rho) n}_{\text{proliferation and death}} + \underbrace{\beta \partial_{yy}^2 n}_{\text{phenotype changes}} \quad y \in x(0,1) \\ \rho(t,x) := \int_0^1 n(t,x,y) dy \quad x \in \Omega \\ \partial_t s - \underbrace{\alpha_s \Delta s}_{\text{diffusion}} = \underbrace{\Sigma(t,x) - \lambda_s s}_{\text{influx and decay}} - \underbrace{\int_0^1 q(s,y) n(t,x,y) dy}_{\text{consumption}} \end{array} \right.$$

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## Main results



Data in the left column from Schuh *et al.*, *Dermatol Ther* 7:187–202, 2017.

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## Immunotherapy in Hematological Malignancies 2023

### Concluding Remarks

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## Immunotherapy in Hematological Malignancies 2023

- **Object of study:** mathematical models for the evolutionary and spatial cellular dynamics implicated in cancer, which are formulated as PDEs.
- **Research method:** develop, study, and use mathematically tractable models that make it possible to achieve more robust conclusions.
- **Outcomes:**
  1. theoretical results that shed light on the mechanisms that underpin cancer development and response to therapy;
  2. new testable biological hypotheses;
  3. novel mathematical problems, methods, and results.

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## Immunotherapy in Hematological Malignancies 2023

Many thanks to my collaborators:

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(RWTH Aachen)

Chiara Villa  
(Sorbonne Université)

Thank you all for your attention

T. Lorenzi (Politecnico di Torino)